

MONOTONICITY, CONVEXITY, AND INEQUALITIES FOR FUNCTIONS INVOLVING GAMMA FUNCTION

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Abstract. In this paper, we study some properties such as the monotonicity, logarithmically complete monotonicity, logarithmic convexity, and geometric convexity, of the combinations of gamma function and power function. The obtained results generalize some related known results for parameters with specific values.

Mathematics subject classification (2020): 33E50, 33E20.

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