Journal of Mathematical Inequalities
Volume 17, Number 3 (2023), 1087–1112
doi:10.7153/jmi-2023-17-71

ASYMPTOTIC PROPERTIES OF STOCHASTIC PREY–PREDATOR MODELS

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Abstract. In this paper, we study three stochastic two-species predator-prey models. We construct stochastic models from deterministic models by introducing three different stochastic perturbations to the growth equations of the prey and predator populations. For the first model, we obtain sufficient conditions for the stochastic model to be asymptotically stable in probability at three different equilibrium points. In addition, using a suitable stochastic Lyapunov method, we study the existence and uniqueness of the solution, the existence of positive recurrence and the ultimate boundedness of the three stochastic systems under certain conditions. we also discuss the global asymptotic stability of the equilibrium point and extinction of the last two stochastic systems. Finally, we provide some numerical simulations to illustrate our mathematical results. We show that stochastic models still retain the desirable stability property of their deterministic counterparts if stochastic perturbations are relatively small.

Mathematics subject classification (2020): 60G15, 60H10.
Keywords and phrases: Asymptotic stability, extinction, ultimately boundedness, positive recurrence.

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