

## THE RELATIONSHIP BETWEEN $r$ -CONVEXITY AND SCHUR-CONVEXITY AND ITS APPLICATION

TAO ZHANG\*, ALATANCANG CHEN, BO-YAN XI AND HUAN-NAN SHI

**Abstract.** In this paper, the relationship between  $r$ -convexity and Schur-convexity is investigated first. Moreover, the  $r$ -convexity and Schur-convexity of a class of functions are studied. As applications, some new inequalities on Minkowski's inequality are established.

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