HARDY–TYPE INEQUALITIES FOR ℓ^p SEQUENCES

LUDOVICK BOUTHAT, JAVAD MASHREGHI AND FRÉDÉRIC MORNEAU-GUÉRIN*

Abstract. We establish three Hardy-type inequalities in which the arithmetic means of a sequence of non-negative real numbers are replaced by the weighted means over nested subsets of the sequence. This work originated from some delicate calculations previously performed for evaluating the norm of infinite L-matrices.

Mathematics subject classification (2020): 26D15, 47A30.
Keywords and phrases: Hardy inequality, arithmetic average, means, ℓ^p spaces.

REFERENCES

[1] RAVI PRAKASH AGARWAL, DONAL O'REGAN, AND SAMIR H. SAKER, *Hardy type inequalities on time scales*, Springer, Cham, 2016.

[2] ALEXANDER A. BALINSKY, WILLIAM DESMOND EVANS, AND ROGER T. LEWIS, *The analysis and geometry of Hardy's inequality*, Universitext, Springer, Cham, 2015.

[3] LUDOVICK BOUTHAT AND JAVAD MASHREGHI, *The norm of an infinite L-matrix*, Operators and Matrices, to appear, pp. 1–12.

[4] THOMAS ARTHUR ALAN BROADBENT, *A Proof of Hardy's Convergence Theorem*, J. London Math. Soc., **3** (4): 242–243, 1928.

[5] EDWARD BRIAN DAVIES, *A review of Hardy inequalities*, In The Mazya anniversary collection, vol. 2 (Rostock, 1998), vol. 110 of Oper. Theory Adv. Appl., pp. 55–67, Birkhäuser, Basel, 1999.

[6] DAVID ERIC EDMUNDS AND WILLIAM DESMOND EVAN, *Hardy operators, function spaces and embeddings*, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2004.

[7] EDWIN BAILEY ELLIOTT, *A Simple Exposition of Some Recently Proved Facts as to Convergency*, J. London Math. Soc., **1** (2): 93–96, 1926.

[8] KARL GRANDJOT, *On Some Identities Relating to Hardy's Convergence Theorem*, J. London Math. Soc., **3** (2): 114–117, 1928.

[9] JACQUES HADAMARD, *Essai sur l'étude des fonctions, données par leur développement de Taylor*, Gauthier-Villars, 1892.

[10] GODFREY HAROLD HARDY, *Notes on some points in the integral calculus XLI: On the convergence of certain integrals and series*, Messenger of Mathematics, **45**: 163–166, 1915.

[11] GODFREY HAROLD HARDY, *Notes on some points in the integral calculus LI: On Hilbert's double-series theorem, and some connected theorems concerning the convergence of infinite series and integrals*, Messenger of Mathematics, **48**: 107–112, 1919.

[12] GODFREY HAROLD HARDY, *Note on a theorem of Hilbert*, Math. Z., **6** (3–4): 314–317, 1920.

[13] GODFREY HAROLD HARDY, *Notes on some points in the integral calculus LX: An inequality between integrals*, Messenger of Mathematics, **54**: 150–156, 1925.

[14] DAVID HILBERT, *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen*, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 1906.

[15] THEODOR KALUZA AND GABOR SZEGÖ, *Uber Reihen Mit Lauter Positiven Gliedern*, J. London Math. Soc., **2** (4): 266–272, 1927.

[16] KONRAD KNOPP, *Uber Reihen Mit Positiven Gliedern*, J. London Math. Soc., **3** (3): 205–211, 1928.

[17] STEVEN GEORGE KRANTZ, *Handbook of complex variables*, Birkhäuser Boston, Inc., Boston, MA, 1999.

[18] KRISTINA KRULIĆ HIMMELREICH, JOSIP PEČARIĆ, AND DORA POKAZ, *Inequalities of Hardy and Jensen. New Hardy type inequalities with general kernels*, vol. 6 of Monographs in Inequalities, Element, Zagreb, 2013.

[19] ALOIS KUFNER, LECH MALIGRANDA, AND LARS-ERIK PERSSON, *The prehistory of the Hardy inequality*, Amer. Math. Monthly, **113** (8): 715–732, 2006.

[20] ALOIS KUFNER, LECH MALIGRANDA, AND LARS-ERIK PERSSON, *The Hardy inequality: About its history and some related results*, Vydavatelský Servis, Plzeň, 2007.

[21] ALOIS KUFNER, LARS-ERIK PERSSON, AND NATASHA SAMKO, *Weighted inequalities of Hardy type*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, second edition, 2017.

[22] EDMUND LANDAU, ISSAI SCHUR, AND GODFREY HAROLD HARDY, *A Note on a Theorem Concerning Series of Positive Terms: Extract from a Letter*, J. London Math. Soc., **1** (1): 38–39, 1926.

[23] NADER MASMOUDI, *About the Hardy inequality*, In An Invitation to Mathematics, 165–180, Springer, 2011.

[24] BOHUMÍR OPIC, AND ALOIS KUFNER, *Hardy-type inequalities*, vol. **219** of Pitman Research Notes in Mathematics Series, Longman Scientific & Technical, Harlow, 1990.

[25] LARS-ERIK PERSSON AND NATASHA SAMKO, *What should have happened if Hardy had discovered this?*, J. Inequal. Appl., **2012**: 1–11, 2012.

[26] MICHAEL RUZHANSKY AND DURVUDKHAN SURAGAN, *Hardy inequalities on homogeneous groups: 100 years of Hardy inequalities*, Springer Nature, 2019.

[27] ISSAI SCHUR, *Bemerkungen zur Theorie der beschränkten Bilinearformen mit unendlich vielen Veränderlichen*, J. Reine Angew. Math., **140**: 1–28, 1911.

[28] HERMANN WEYL, *Singuläre Integralgleichungen*, Math. Ann., **66** (3): 273–324, 1908.

[29] HERMANN WEYL, *Singuläre Integralgleichungenmit besonder Berücksichtigung des Fourierschen Integraltheorems*, PhD thesis, Georg-August-Universität Göttingen, 1908.

[30] FRIEDRICH WILHELM WIENER, *Elementarer Beweis eines Reihensatzes von Herrn Hilbert*, Math. Ann., **68** (3): 361–366, 1910.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com