HERMITE—HADAMARD–TYPE INEQUALITIES FOR *DIFFERENTIABLE MULTIPLICATIVE m–PREINVEXITY AND (s,m)–PREINVEXITY VIA THE MULTIPLICATIVE TEMPERED FRACTIONAL INTEGRALS

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Abstract. In virtue of the conception of the multiplicative tempered fractional integrals, put forward by Fu et al. in the published article [AIMS Math., 6 (7): 7456–7478, 2021], we present a fractional integral identity for *differentiable functions. Based upon it, we develop several inequalities of Hermite–Hadamard type in association with *differentiable multiplicative m-preinvexity and (s,m)-preinvexity.

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