MORE ACCURATE FORM OF HALF–DISCRETE HILBERT–TYPE INEQUALITY WITH A GENERAL KERNEL

MINGHUI YOU*, WEI SONG AND FEI DONG

Abstract. In this work, by constructing a new kernel function in general form which includes both the homogeneous and the non-homogeneous cases, a half-discrete Hilbert-type inequality involving the newly constructed kernel function is established. Additionally, the equivalent Hardy-type inequalities are considered, and all the constant factors in the newly obtained inequalities are proved to be the best possible. Furthermore, by specializing the kernel function and introducing some special functions such as Beta function and Gamma function, some existing results and new examples are presented at the end of the paper.

Mathematics subject classification (2020): 26D15, 41A17.
Keywords and phrases: Hilbert-type inequality, general kernel function, more accurate form, Beta function, Gamma function.

REFERENCES

[1] W. ABUELELA, *A generalization of a half-discrete Hilbert's inequality* , J. Egyptian Math. Soc., **24**, (2016), 171–174.

[2] I. BRNETIĆ AND J. PEČARIĆ, *Generalization of Hilbert's integral inequality*, Math. Inequal. Appl., **7**, 2 (2004), 199–205.

[3] T. BATBOLD, M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *Further development of Hilbert-type inequalities*, Element Press, Zagreb, 2017.

[4] Q. CHEN, Y. HONG AND B. C. YANG, *A more accurate extended Hardy-Hilbert's inequality with parameters*, J. Math. Inequal., **16**, 3 (2022), 1075–1089.

[5] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge Univ. Press, London, 1952.

[6] Y. HONG, J. Q. LIAO, B. C. YANG AND Q. CHEN, *A class of Hilbert-type multiple integral inequalities with the kernel of generalized homogeneous function and its applications*, J. Inequal. Appl., (2020), https://doi.org/10.1186/s13660-020-02401-0.

[7] Y. HONG, B. HE AND M. FENG, *Equivalent conditions of optimal half discrete Hilbert type multiple integral inequalities with quasi homogeneous kernel and applications*, J. Math. Inequal. **16**, 1 (2022), 83–95.

[8] B. HE, B. C. YANG AND Q. CHEN, *A new multiple half-discrete Hilbert-type inequality with parameters and a best possible constant factor*, Mediterr. J. Math., **12**, (2015), 1227–1244.

[9] M. KRNIĆ, J. PEČARIĆ, I. PERIĆ AND P. VUKOVIĆ, *Recent advances in Hilbert-type inequalities*, Element Press, Zagreb, 2012.

[10] M. KRNIĆ AND J. PEČARIĆ, *Extension of Hilbert's inequality*, J. Math. Anal. Appl., **324**, (2006), 150–160.

[11] M. KRNIĆ AND J. PEČARIĆ, *On some general Hilbert's inequalities with non-conjugate parameters*, Rend. Circ. Mat. Palermo., **56**, 2 (2007), 219–234.

[12] M. KRNIĆ AND J. PEČARIĆ, *General Hilbert's and Hardy's inequalities*, Math. Inequal. Appl., **8**, (2005), 29–51.

[13] M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *Discrete Hilbert-type inequalities with general homogeneous kernels*, Rend. Circ. Mat. Palermo., **60**, 1 (2011), 161–171.

[14] M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *A unified treatment of half-discrete Hilbert-type inequalities with a homogeneous kernel*, Mediterr. J. Math., **10**, (2013), 1697–1716.

[15] M. KRNIĆ AND J. PEČARIĆ, *A Hilbert inequality and an Euler-Maclaurin summation formula*, ANZIAM J., **48**, 2007, 419–431.

[16] J. C. KUANG AND L. DEBNATH, *On new generalizations of Hilbert's inequality and their applications*, J. Math. Anal. Appl., **245**, (2000), 248–265.

[17] M. T. RASSIAS AND B. C. YANG, *A Hilbert-type integral inequality in the whole plane related to the hypergeometric function and the beta function* , J. Math. Anal. Appl., **428**, 4 (2015), 1286–1308.

[18] M. T. RASSIAS AND B. C. YANG, *On an equivalent property of a reverse Hilbert-type integral inequality related to the extended Hurwitz-Zeta function*, J. Math. Inequal., **13**, 2 (2019), 315–334.

[19] M. T. RASSIAS, B. C. YANG AND A. RAIGORODSKII, *Two kinds of the reverse Hardy-type integral inequalities with the equivalent forms related to the extended Riemann Zeta function*, Appl. Anal. Discrete Math., **12**, (2018), 273–296.

[20] A. Z. WANG AND B. C. YANG, *A new more accurate half-discrete Hilbert-type inequality*, J. Inequal. Appl., (2012), https://doi:10.1186/1029-242X-2012-260.

[21] B. C. YANG, *On Hilbert's integral inequality*, J. Math. Anal.Appl., **220**, (1998), 778–785.

[22] B. C. YANG, I. BRNETIC, M. KRNIĆ AND J. PEČARIĆ, *Generalization of Hilbert and Hardy-Hilbert integral inequalities*, Math. Inequal. Appl., **8**, 2 (2005), 259–272.

[23] B. C. YANG, *On an extension of Hilbert's integral inequality with some parameters*, Aust. J. Math. Anal. Appl., **1**, 1 (2004), 1–8.

[24] B. C. YANG, *The norm of operator and Hilbert-type inequalities*, Science Press, Beijing, 2009.

[25] B. C. YANG, *A half-discrete Hilbert's inequality*, J. Guangdong Univ. Educ., **31**, 3 (2011), 1–7.

[26] M. H. YOU, *On a new discrete Hilbert-type inequality and application*, Math. Inequal. Appl. **18**, 4 (2015), 1575–1578.

[27] M. H. YOU AND X. SUN, *On a Hilbert-type inequality with the kernel involving extended Hardy operator*, J. Math. Inequal., **15**, 3 (2021), 1239–1253.

[28] M. H. YOU AND Y. GUAN, *On a Hilbert-type integral inequality with non-homogeneous kernel of mixed hyperbolic functions*, J. Math. Inequal., **13**, 4 (2015), 1197–1208.

[29] M. H. YOU, F. DONG AND Z. H. HE, *A Hilbert-type inequality in the whole plane with the constant factor related to some special constants*, J. Math. Inequal. **14**, 1 (2022), 35–50.

[30] Z. X. WANG AND D. R. GUO, *Introduction to Special Functions*, Higher Education Press, Beijing, 2012.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com