A GENERALIZED REFINEMENT OF YOUNG'S INEQUALITY

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Abstract. In this paper, we mainly give a generalized refinement of Young's inequality due to Yang and Wang [J. Math. Inequal., 17 (2023), 205–217]. More precisely, we show that

$$\frac{(a\nabla_v b)^m - K(h,2)^{mv}(a\sharp_v b)^m}{(a\nabla_\tau b)^m - K(h,2)^{m\tau}(a\sharp_\tau b)^m} \geqslant \frac{v(1-v)}{\tau(1-\tau)},$$

where $0 < v \leqslant \tau < \frac{1}{2}$, $m \in \mathbb{N}^+$, $a > b > 0$, $K(h,2) = \frac{(h+1)^2}{4h}$ and $h = \frac{b}{a}$. As applications, we obtain some inequalities for operator, Hilbert-Schmidt norm and trace class norm.

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