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STABILITY OF A NEW ADDITIVE FUNCTIONAL INEQUALITY IN BANACH SPACES

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Abstract. We propose and solve a new type of additive functional inequality. We also obtain the Hyers-Ulam stability of such functional inequality in a complex Banach space by using two different approaches.

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