Journal of Mathematical Inequalities
Volume 18, Number 1 (2024), 51–67
doi:10.7153/jmi-2024-18-04

MAJORIZATION TYPE INEQUALITIES VIA 4–CONVEX FUNCTIONS

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Abstract. The main aim of this paper is to prove several majorization type inequalities using Green and 4-convex functions. First of all, we drive generalized majorization inequality for arbitrary n-tuples and real weights. Further, we explore the inequality for majorized tuples, weighted majorization theorems given by Fuchs, Dragomir and Maligranda et al. For deriving another generalized majorization inequality, we use a simple form of Jensen's inequality, and by similar fashion we apply classical earlier majorization theorems for further elaborations of generalized inequality. Several applications of information theory are discussed at the end of the article.

Mathematics subject classification (2020): 26D15, 94A17, 94A15.
Keywords and phrases: Majorization inequality, Green functions, information theory.

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