Journal of Mathematical Inequalities
Volume 18, Number 1 (2024), 103–126
doi:10.7153/jmi-2024-18-07

GENERALIZED WEIGHTED HARDY'S INEQUALITIES WITH COMPACT PERTURBATIONS

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Abstract. Let Ω be a bounded domain of $\mathbf{R}^N$ $(N \geqslant 1)$ with boundary of class C^2. In the present paper we shall study a variational problem relating the weighted Hardy inequalities with sharp missing terms established in [8]. As weights we treat non-doubling functions of the distance $\delta(x) = \operatorname{dist}(x, \partial\Omega)$ to the boundary $\partial\Omega$.

Mathematics subject classification (2020): Primary 35J70; Secondary 35J60, 34L30, 26D10.
Keywords and phrases: Weighted Hardy's inequalities, nonlinear eigenvalue problem, weak Hardy property, p-Laplace operator with weights.

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