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BOUNDS FOR THE α–ADJACENCY ENERGY OF A GRAPH

REZWAN UL SHABAN, MUHAMMAD IMRAN AND HILAL A. GANIE

Abstract. For the adjacency matrix $A(G)$ and diagonal matrix of the vertex degrees $D(G)$ of a simple graph G, the $A_\alpha(G)$ matrix is the convex combinations of $D(G)$ and $A(G)$, and is defined as $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$, for $0 \leqslant \alpha \leqslant 1$. Let $\rho_1 \geqslant \rho_2 \geqslant \ldots \geqslant \rho_n$ be the eigenvalues of $A_\alpha(G)$ (which we call α-adjacency eigenvalues of the graph G). The generalized adjacency energy also called α-adjacency energy of the graph G is defined as $E^{A_\alpha}(G) = \sum_{i=1}^{n} \left|\rho_i - \alpha\overline{d}\right|$, where $\overline{d} = \frac{2m}{n}$ is the average vertex degree, m is the size and n is the order of G. The α-adjacency energy of a graph G merges the theory of energy (adjacency energy) and the signless Laplacian energy, as $E^{A_0}(G) = \mathscr{E}(G)$ and $2E^{A_{\frac{1}{2}}}(G) = QE(G)$, where $\mathscr{E}(G)$ is the energy and $QE(G)$ is the signless Laplacian energy of G. In this paper, we obtain some new upper and lower bounds for the generalized adjacency energy of a graph, in terms of different graph parameters like the vertex covering number, the Zagreb index, the number of edges, the number of vertices, etc. We characterize the extremal graphs attained these bounds.

Mathematics subject classification (2020): 05C50, 05C12, 15A18.
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