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ON FUZZY SPECTRAL RADII FOR FUZZY BOUNDED OPERATORS WITH APPLICATION TO FUZZY VOLTERRA OPERATOR

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Abstract. This paper's aim is to extend and generalize the classic results regarding spectral radii and the corresponding resolvent sets for some different classes of bounded operators acting on fuzzy normed spaces. In this context, the fuzzy norm definition introduced giving shape to a new topology for a fuzzy space, namely a fuzzy topology, also gives the opportunity to study the behavior of various types of operators defined between fuzzy normed spaces, along with their spectral properties. There are several definitions for resolvent sets and consequently, several corresponding definitions of spectral radii that will be considered in this work, since these are non-equivalent ways of defining such notions. Spectral radii are calculated for a fuzzy Volterra type operator acting between fuzzy normed spaces.

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