INEQUALITIES FOR FUNCTIONS CONVEX ON THE COORDINATES WITH APPLICATIONS TO JENSEN AND HERMITE–HADAMARD TYPE INEQUALITIES, AND TO NEW DIVERGENCE FUNCTIONALS

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Abstract. In this paper we show that inequalities for functions convex on the coordinates can be derived from inequalities for convex functions defined on real intervals, and essentially only this method works. As applications, we show how our result works for the Jensen's and Hermite-Hadamard inequalities for functions convex on the coordinates. Finally, we extend the classical notion of f-divergence functional to functions convex on the coordinates, and as a further application of our main result, we study the refinement of a basic inequality corresponding to the new divergence.

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