B_σ TYPE MIXED MORREY SPACES AND THEIR APPLICATIONS

YICHUN ZHAO AND JIANG ZHOU *

Abstract. This paper defines and studies the B_σ type mixed Morrey spaces. Furthermore, we introduce the predual spaces of B_σ type mixed Morrey spaces to establish the extrapolation theorem. As applications of extrapolation theory, we character the BMO space in different ways. On the one hand, we character the BMO spaces by the new John-Nirenberg inequality in terms of B_σ type mixed Morrey spaces. On the other hand, we establish the characterization of BMO spaces by the boundedness of commutators of the singular integral operator on B_σ type mixed Morrey spaces.

Mathematics subject classification (2020): 42B20, 42B25, 42B35.

Keywords and phrases: Mixed Morrey space, duality, extrapolation, John-Nirenberg inequality, characterization.

REFERENCES

[1] A. BENEDEK AND R. PANZONE, *The space L_p with mixed norm*, Duke Math. J., **28**, no. 3, 301–324, 1961.

[2] A. BEURLING, *Construction and analysis of some convolution algebras*, Ann. Inst. Fourier, **14**, no. 2, 1–32, 1964.

[3] V. I. BURENKOV, V. S. GULIEV, AND G. V. GULIEV, *Necessary and sufficient conditions for the boundedness of the fractional maximal operator in local Morrey-type spaces*, Doklady Math., **74**, no. 1, 540–544, 2006.

[4] S. CHANILLO, *A note on commutators*, Indiana U. Math. J., **31**, no. 1, 7–16, 1982.

[5] Y. CHEN, K. LAU, *Some new classes of Hardy spaces*, J. Funct. Anal., **84**, no. 2, 255–278, 1989.

[6] R. COIFMAN, R. ROCHBERG AND G. WEISS, *Factorization theorems for Hardy spaces in several variables*, Ann. Math., **84**, no. 2, 611–635, 1976.

[7] J. L. R. D. FRANCIA, *Factorization and extrapolation of A_p weights*, Bull. Amer. Ath. Soc, no. 3, 533–547, 1984.

[8] J. GARCÍA-CUERVA, *Hardy spaces and Beurling algebras*, J. Lond. Math. Soc., **2**, no. 3, 499–513, 1989.

[9] L. GRAFAKOS, *Modern fourier analysis*, vol. 250, Springer, 2009.

[10] L. GRAFAKOS, *Classical Fourier Analysis*, vol. 249, Springer, 2009.

[11] K. P. HO, *John-Nirenberg inequalities on Lebesgue spaces with variable exponents*, Taiwan. J. Math., **18**, no. 4, 1107–1118, 2014.

[12] K. P. HO, *Extrapolation, John-Nirenberg inequalities and characterizations of BMO in terms of Morrey type spaces*, Rev. Mat. Complu., **30**, no. 3, 1–19, 2017.

[13] K. P. HO, *Intrinsic square functions on Morrey and Block spaces with variable exponents*, Bull. Malays. Math. Sci. Soc., **40**, 995–1010, 2017.

[14] K. P. HO, *Boundedness of operators and inequalities on Morrey Banach spaces*, Publ. Res. Inst. Math. Sci., **58**, no. 3, 551–577, 2022.

[15] K. P. HO, *Extrapolation to mixed-norm spaces and applications*, Acta Comment. Univ. Tartu. Math., **25**, no. 2, 281–296, 2021.

[16] K. P. HO, *Sublinear operators on Herz-Hardy spaces with variable exponents*, Math. Nachr., **295**, no. 5, 876–889, 2022.

[17] L. HÖRMANDER, *Estimates for translation invariant operators in L^p spaces*, Acta Math., **104**, 93–140, 1960.

[18] M. IZUKI, T. NOI AND Y. SAWANO, *The John–Nirenberg inequality in ball Banach function spaces and application to characterization of BMO*, J. Ineq. Appl., **2019**, no. 1, 1–11, 2019.

[19] S. JANSON, *Mean oscillation and commutators of singular integral operators*, Ark. Mat., **16**, no. 1–2, 263–270, 1978.

[20] F. JOHN AND L. NIRENBERG, *On functions of bounded mean oscillation*, Commu. Pure Appl. Math., **14**, no. 3, 415–426, 1961.

[21] V. KOKILASHVILI, A. MESKHI AND M. A. RAGUSA, *Weighted extrapolation in grand Morrey spaces and applications to partial differential equations*, Rend. Lincei, **30**, no. 1, 67–92, 2019.

[22] Y. K. FURUYA AND K. MATSUOKA, *Fractional integrals on B_σ weighted Morrey spaces*, Math. Inequal. Appl., **19**, no. 3, 969–980, 2016.

[23] Y. K. FURUYA AND K. MATSUOKA, E. NAKAI AND Y. SAWANO, *Applications of Littlewood-Paley theory for $\dot{B}_\sigma$ Morrey spaces to the boundedness of integral operators*, J. Funct. Spaces, **13**, no. 3, 1–21, 2013.

[24] Y. K. FURUYA AND K. MATSUOKA, E. NAKAI AND Y. SAWANO, *Integral operators on B_σ Morrey Campanato spaces*, Rev. Mat. Complu., **26**, 1–32, 2013.

[25] S. LU AND D. YANG, *The central BMO spaces and Littlewood-Paley operators*, Approx. Theory Appl., **11**, no. 3, 72–94, 1995.

[26] S. LU, D. YANG, *The Littlewood-Paley function and φ transform characterizations of a new Hardy space HK_2 associated with the Herz space*, Studia Math., **101**, 285–298, 1992.

[27] W. LU AND J. ZHOU, *Some estimates of multi-sublinear operators and commutators on mixed λ-central Morrey spaces*, Ann. Funct. Anal., **14**, no. 1, 39, 1–26, 2023.

[28] K. MATSUOKA, *B_σ Campanato estimates for commutators of Calderón-Zygmund operators*, Ann. Inst. Statist. Math., **58**, no. 1, 131–151, 2014.

[29] K. MATSUOKA AND E. NAKAI, *Fractional integral operators on $B^{p,\lambda}$ with Morrey-Campanato norms*, Banach Center Publ., **92**, 249–264, 2011.

[30] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc., **43**, no. 1, 126–166, 1938.

[31] B. MUCKENHOUPT AND R. L. WHEEDEN, *Norm inequalities for the Littlewood-Paley function*, Trans. Amer. Math. Soc., **191**, 95–111, 1974.

[32] E. NAKAI, *Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces*, Math. Nachr., **166**, no. 1, 95–103, 1994.

[33] E. NAKAI AND T. SOBUKAWA, *B_w^u function spaces and their interpolation*, Mathematics, **39**, no. 2, 483–516, 2016.

[34] Z. NIERAETH, *Extrapolation in general quasi-Banach function spaces*, J. Funct. Anal., **285**, no. 10, 110–130, 2023.

[35] T. NOGAYAMA, *Boundedness of commutators of fractional integral operators on mixed Morrey spaces*, Integr. Transf. Spec. Funct., **30**, no. 10, 790–816, 2019.

[36] T. NOGAYAMA, *Mixed Morrey spaces*, Positivity, **23**, 961–1000, 2019.

[37] T. NOGAYAMA, T. ONO, D. SALIM, AND Y. SAWANO, *Atomic decomposition for mixed Morrey spaces*, J. Geom. Anal., **38**, 9338–9365, 2021.

[38] Y. SAWANO AND H. YOSHIDA, *A predual of a predual of B_σ and its applications to commutators*, Sci. China (Mathematics), **61**, no. 008, 1437–1472, 2018.

[39] M. SHI AND J. ZHOU, *Some estimates of operators on local mixed Morrey-type spaces*, Indian J. Pure Appl. Math., 1–20, 2023.

[40] S. SAMKO, *Variable exponent Herz spaces*, Mediterranean J. Math., **10**, no. 4, 2007–2025, 2013.

[41] M. WEI, *Boundedness criterion for some integral operators on generalized mixed Morrey spaces and generalized mixed Hardy Morrey spaces*, Banach J. Math. Anal., **16**, no. 1, 1–26, 2022.

[42] M. WEI, *Estimates for weighted Hardy-Littlewood averages and their commutators on mixed central Morrey spaces*, J. Math. Ineq., **16**, no. 2, 659–669, 2022.

[43] M. WEI, *Linear operators and their commutators generated by Calderón–Zygmund operators on generalized Morrey spaces associated with ball Banach function spaces*, Positivity, **26**, no. 5, 84, 1–26, 2022.

[44] M. WEI, *Extrapolation for weighted product Morrey spaces and some applications*, Potential Anal., 1–28, 2023.

[45] M. WEI, *A characterization of $\mathrm{CMO}^{\vec{q}}$ via the commutator of Hardy-type operators on mixed Herz spaces*, Appl. Anal., **101**, no. 16, 1–16, 2021.

[46] M. WEI, *Block decomposition for Herz spaces associated with ball Banach function spaces*, J. Math. Ineq., **17**, no. 2, 641–652, 2023.

[47] H. ZHANG AND J. ZHOU, *The boundedness of fractional integral operators in local and global mixed Morrey type spaces*, Positivity, **26**, no. 1, 26, 1–26, 2022.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com