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RELATIVE GROWTH OF A COMPLEX POLYNOMIAL WITH RESTRICTED ZEROS

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Abstract. Let $p(z)$ be a polynomial of degree n with zero of multiplicity s at the origin and the remaining zeros be in $|z| \geqslant k$ or in $|z| \leqslant k$, $k > 0$. In this paper, we investigate the relative growth of a polynomial $p(z)$ with respect to two circles $|z| = r$ and $|z| = R$ and obtain inequalities about the dependence of $|p(rz)|$ on $|p(Rz)|$, where $|z| = 1$, for $0 < r \leqslant R \leqslant k$ or $0 < k \leqslant R \leqslant r$ while taking into account the placement of the zeros of the underlying polynomial. Our results improve as well as generalize certain well-known polynomial inequalities. Some numerical examples are also given in order to illustrate and compare graphically the obtained inequalities with some recent results.

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