

A SHARP MID-POINT TYPE INEQUALITY

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Abstract. This paper deals with a sharp version of mid-point type inequality in connection with fractional integrals of real valued absolutely continuous functions as a generalization and refinement of non-sharp classical mid-point inequality which is presented by the Riemann integrals of differentiable real valued functions whose derivative absolute values are convex. Some special functions, numerical means and a mid-point type formula are considered to discuss about some applications of main results.

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