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NORMAL APPROXIMATION FOR A RANDOMLY INDEXED BRANCHING PROCESS

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Abstract. Consider a supercritical Galton-Watson process $\{Z_n, n \geqslant 0\}$ and an independent renewal process $\{N(t), t \geqslant 0\}$, one-term Edgeworth expansions and Cramér type moderate deviations for the logarithm of $Z_{N(t)}$ are developed. Examples are also given to illustrate our results.

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