Journal of Mathematical Inequalities
Volume 18, Number 2 (2024), 401–442
doi:10.7153/jmi-2024-18-23

JENSEN–MARSHALL–KY FAN–TYPE INEQUALITIES AND THEIR APPLICATIONS IN BUSINESS PROFIT MANAGEMENT MODEL

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Abstract. This paper will introduce the theory of ϕ-Jensen coefficient. By means of the functional analysis, linear algebra, discrete mathematics and inequality theories with proper hypotheses, the Jensen-type inequality, Marshall-type inequality and the Ky Fan-type inequality are obtained as follows:

$$\widehat{f}|_{\phi} \geqslant \widehat{g}|_{\phi},\ \widetilde{f}_{\gamma} \geqslant \widetilde{g}_{\gamma} \text{ and } \widehat{\varphi}_{\gamma} \geqslant \widehat{(1-\varphi)}_{\gamma},$$

respectively, as well as we also displayed the applications of our main results in business profit management model, and some conditions such that $\mathbf{p} \prec_{\gamma} \mathbf{e}$ or $\mathbf{p} \backsim_{\gamma} \mathbf{e}$ hold are obtained, where $\mathbf{p}$ is the profit function and $\mathbf{e}$ is the cost function.

Mathematics subject classification (2020): 26D15, 26E60, 62J10.
Keywords and phrases: ϕ-Jensen coefficient, Jensen inequality, Marshall inequality, Ky Fan inequality, business profit management model.

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