Journal of Mathematical Inequalities
Volume 18, Number 2 (2024), 519–531
doi:10.7153/jmi-2024-18-28

SOME REFINEMENTS OF YOUNG TYPE INEQUALITIES

CHANGSEN YANG AND GEGE ZHANG

Abstract. In this paper, we give some new improvements and reverse improvements of Young type inequalities. The conclusion proved by Yang and Wang [J. Math. Inequal., 17 (2023), 205–217] involved the monotonicity of $\frac{K(x,2)^{\upsilon}(x^{\upsilon})-(1-\upsilon+\upsilon x)}{\upsilon}$, where $K(x,2)=\frac{(x+1)^2}{4x}$, $x>0$ and $\frac{1}{2}\leqslant\upsilon\leqslant 1$. This article demonstrates the monotonicity of $\frac{M_{\upsilon}^{\upsilon}(x)(x^{\upsilon})-(1-\upsilon+\upsilon x)}{\upsilon}$, where $M_{\upsilon}(x)=1+\upsilon(1-\upsilon)\frac{(x-1)^2}{x}$, $x\geqslant 1$ and $\frac{1}{2}\leqslant\upsilon\leqslant\frac{3}{4}$. And this implies a main conclusion as follows:

$$\frac{M_{\upsilon}^{\upsilon}(h)a\sharp_{\upsilon}b-a\nabla_{\upsilon}b}{M_{\tau}^{\tau}(h)a\sharp_{\tau}b-a\nabla_{\tau}b}\leqslant\frac{\upsilon}{\tau},$$

where $\frac{1}{2}\leqslant\upsilon<\tau\leqslant\frac{3}{4}$, $b\geqslant a>0$, $M_{\upsilon}(h)=1+\upsilon(1-\upsilon)\frac{(h-1)^2}{h}$ and $h=\frac{b}{a}$.

Furthermore, we can get some related results about operator, Hilbert-Schmidt norm, trace norm by these scalars results.

Mathematics subject classification (2020): 47A63, 47A64, 15A60.

Keywords and phrases: Arithmetic-geometric-harmonic, Kantorovich constant, Young-type inequalities.

REFERENCES

[1] S. DRAGOMIR, *A note on Young's inequality*, Rev. R. Acad. Cienc. Exactas. Fis. nat. Ser. A. Math., **111** (2017), 349–354.

[2] S. FURUICHI, *Further improvement of Young inequality*, Rev. R. Acad. Cienc. Exactas. Fis. nat. Ser. A. Math., **113** (2019), 255–266.

[3] A. GHAZANFARI, S. MALEKINEJAD AND S. TALEBI, *Some new inequalities involving Heinz operator means*, J. Math. Anal., **7** (2016), 147–155.

[4] F. HIAI AND H. KOSAKI, *Means for matrices and comparison of their norms*, India Univ. Math. J., **48**, 3 (1999), 899–936.

[5] H. KOSAKI, *Positive definiteness of functions with applications to operator norm inequalities*, Mem. Amer. Math. Soc., **212** (no. 997), 2011.

[6] W. LIAO, J. WU, J. ZHAO, *New versions of reverse Young and Heinz mean inequalities with the Kantorovich constant*, Taiwanese J. Math., **19** (2015), 467–479.

[7] J. PEČARIĆ, T. FURUTA, J. HOT, Y. SEO, *Mond-Pečarić Method in Operator Inequalities*, Element, Zagreb, 2005.

[8] Y. REN, *Some results of Young-type inequalities*, Rev. R. Acad. Cienc. Exactas. Fis. nat. Ser. A. Math., **114** (2020), 114–143.

[9] H. ZUO, Y. LI, *Some refinements of Young type inequalities*, J. Math. Inequal., **16** (2022), 1169–1178.

[10] H. ZUO, G. SHI AND M. FUJII, *Refined Young inequality with Kantorovich constant*, J. Math. Inequal., **5**, 4 (2011), 551–556.

[11] C. YANG AND Y. LI, *A new Young type inequality involoving Heinz mean*, Filomat, **34**, 11 (2020), 3639–3654.

[12] C. YANG, Z. WANG, *Some new improvements of Young's inequalities*, J. Math. Inequal., **17**, 1 (2023), 205–217.

Paper JMI-18-28