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SIX CLASSES OF NONLINEAR TWO–DIMENSIONAL SYSTEMS OF DIFFERENCE EQUATIONS WHICH ARE SOLVABLE

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Abstract. We present six classes of nonlinear two-dimensional systems of difference equations which are solvable. Some methods for finding their general solutions are described in detail.

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