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REVERSE OSTROWSKI'S TYPE WEIGHTED INEQUALITIES FOR CONVEX FUNCTIONS ON LINEAR SPACES WITH APPLICATIONS

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Abstract. In this paper we provide several upper and lower bounds for the Ostrowski difference

$$\int_0^1 f((1-t)x+ty)\,w(t)\,dt-\left(\int_0^1 w(t)\,dt\right)f((1-\lambda)x+\lambda y),$$

where $f:C\to\mathbb{R}$ is a convex function, C is a convex subset of a vector space X and w is integrable and nonnegative a.e. on $[0,1]$. A perturbed version under some natural assumptions on the weight function w is also considered. These results are then employed to obtain several weighted integral inequalities for norms and semi-inner products. The particular case of inner product spaces is analyzed and refinements of the weighted integral midpoint inequality for norms are provided.

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