ESTIMATES FOR THE NORM OF THE SPHERICAL MAXIMAL OPERATOR ON FINITE GRAPHS

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Abstract. For a simple, finite, and connected graph G, the spherical maximal operator is defined as

$$\mathcal{M}_G^\circ h(t) = \sup_{r \geqslant 0} \frac{1}{|S(t,r)|} \sum_{u \in S(t,r)} |h(u)|,$$

where $S(t,r) = \{w \in V \mid d_G(w,t) = r\}$ is the sphere with center at t and having radius r. In this paper, we consider the spherical maximal operator $\mathcal{M}_G^\circ$ on ℓ^p spaces and calculate the $\|\mathcal{M}_G^\circ\|_{\ell^p}$ for $0 < p \leqslant 1$ and estimate the $\|\mathcal{M}_G^\circ\|_{\ell^p}$ for $1 < p < \infty$, when G is K_m. Furthermore, We establish the maximum and minimum bounds for the spherical maximum operator on finite graphs and indicate the graphs that achieve these bounds.

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