OPERATOR INEQUALITIES VIA THE TRIANGLE INEQUALITY

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Abstract. This article improves the triangle inequality for complex numbers, using the Hermite-Hadamard inequality for convex functions. Then, applications of the obtained refinement are presented to include some operator inequalities. The operator applications involve numerical radius inequalities and operator mean inequalities.

REFERENCES

[1] T. ANDO AND F. HIAI, *Operator log-convex functions and operator means*, Math. Ann. **350** (2011), 611–630.

[2] M. BAKHERAD AND K. SHEBRAWI, *Upper bounds for numerical radius inequalities involving off-diagonal operator matrices*, Ann. Funct. Anal. **9** (3) (2018), 297–309.

[3] P. BHUNIA, A. BHANJA, S. BAG, AND K. PAUL, *Bounds for the Davis–Wielandt radius of bounded linear operators*, Ann. Funct. Anal. **12** (2021), Article 18.

[4] A. BOURHIM AND M. MABROUK, *Numerical radius and product of elements in C* -algebras*, Linear Multilinear Algebra **65** (6) (2017), 1108–1116.

[5] S. S. DRAGOMIR, J. E. PEČARIĆ AND L. E. PERSSON, *Properties of some functionals related to Jensen's inequality*, Acta Math. Hungar. **70** (1996), 129–143.

[6] S. FURUICHI AND H. R. MORADI, *Advances in mathematical inequalities*, De Gruyter, 2020.

[7] S. FURUICHI, H. R. MORADI AND M. SABABHEH, *Some inequalities for interpolational operator means*, J. Math. Inequal. **15** (1) (2021), 107–116.

[8] H. L. GAU, K. Z. WANG, AND P. Y. WU, *Numerical radii for tensor products of matrices*, Linear Multilinear Algebra **63** (10) (2015), 1916–1936.

[9] I. H. GÜMÜŞ, H. R. MORADI, AND M. SABABHEH, *More accurate operator means inequalities*, J. Math. Anal. Appl. **465** (1) (2018), 267–280.

[10] M. HAJMOHAMADI, R. LASHKARIPOUR, AND M. BAKHERAD, *Some generalizations of numerical radius on off-diagonal part of* 2×2 *operator matrices*, J. Math. Inequal. **12** (2) (2018), 447–457.

[11] O. HIRZALLAH, F. KITTANEH, AND K. SHEBRAWI, *Numerical radius inequalities for certain* 2×2 *operator matrices*, Int. Eq. Oper. Theory **71** (2011), 129–147.

[12] T. KATO, *Notes on some inequalities for linear operators*, Math. Ann. **125** (1952), 208–212.

[13] E. KIKIANTY, *Hermite-Hadamard inequality in the geometry of Banach spaces*, PhD thesis, Victoria University, 2010.

[14] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math. **158** (1) (2003), 11–17.

[15] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math. **168** (1) (2005), 73–80.

[16] F. MITROI, *About the precision in Jensen–Steffensen inequality*, Ann. Univ. Craiova Math. Comput. Sci. Ser. 37 (4) (2010), 73–84.

[17] H. R. MORADI, S. FURUICHI, Z. HEYDARBEYGI AND M. SABABHEH, *Revisiting the Grüss inequality*, Oper. Matrices **15** (4) (2021), 1379–1392.

[18] M. S. MOSLEHIAN AND M. SATTARI, *Inequalities for operator space numerical radius of* 2×2 *block matrices*, J. Math. Phys. **57** (1) (2016), Article 015201.

[19] M. SABABHEH, *Improved Jensen's inequality*, Math. Inequal. Appl. **20** (2) (2017), 389-403.

[20] M. SABABHEH, *Means refinements via convexity*, Mediterr. J. Math. **14** (2017), Article 125.

[21] M. SABABHEH, *On the matrix harmonic mean*, J. Math. Inequal. **12** (4) (2018), 901–920.

[22] M. SABABHEH, S. FURUICHI, Z. HEYDARBEYGI, AND H. R. MORADI, *On the arithmetic-geometric mean inequality*, J. Math. Inequal. **15** (3) (2021), 1255–1266.

[23] M. SABABHEH, S. FURUICHI AND H. R. MORADI, *Improved matrix inequalities using radical convexity*, J. Math. Inequal. **17** (1) (2023), 349–364.

[24] M. SABABHEH, S. FURUICHI, S. SHEYBANI AND H. R. MORADI, *Singular values inequalities for matrix means*, J. Math. Inequal. **16** (1) (2022), 169–179.

[25] M. SABABHEH AND H. R. MORADI, *More accurate numerical radius inequalities*, Linear Multilinear Algebra. **69** (10) (2021), 1964–1973.

[26] S. SHEYBANI, M. SABABHEH, AND H. R. MORADI, *Weighted inequalities for the numerical radius*, Vietnam J. Math. **51** (2) (2023), 363–377.

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