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CHARACTERIZATIONS OF SLICE BESOV–TYPE AND SLICE TRIEBEL—LIZORKIN–TYPE SPACES AND APPLICATIONS

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Abstract. Let $\alpha \in \mathbb{R}$, $\tau \in [0,\infty)$, $q \in (0,\infty]$ and t, r, $p \in (0,\infty)$. In this paper, we introduce the slice Besov-type space $(\dot{B}E^{\alpha,\tau}_{r,p,q})(\mathbb{R}^n)$ and the slice Triebel–Lizorkin-type space $(\dot{F}E^{\alpha,\tau}_{r,p,q})(\mathbb{R}^n)$, and establish their φ-transform characterizations in the sense of Frazier and Jawerth. The embedding properties, characterizations via the Peetre maximal function, the Lusin area function, smooth atomic and molecular decompositions of these spaces are also obtained. As applications, we obtain the boundedness on these spaces of Fourier multipliers with symbols satisfying some generalized Hörmander condition.

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