FRACTIONAL INTEGRAL OPERATORS ON GRAND MORREY SPACES AND GRAND HARDY–MORREY SPACES

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Abstract. This paper establishes the mapping properties of the fractional integral operators on the grand Morrey spaces and the grand Hardy-Morrey spaces defined on the Euclidean spaces. We obtain our results by refining the Rubio de Francia extrapolation method as the existing extrapolation method cannot be directly applied to the grand Morrey spaces. This method also yields the mapping properties of nonlinear operators. In particular, we establish the Sobolev embedding, the Poincaré inequality and the mapping properties of the fractional geometric maximal functions on the grand Morrey spaces.

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