SHARPENING OF INEQUALITIES CONCERNING POLYNOMIALS

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Abstract. Let $P(z) = a_n \prod_{j=1}^{n} (z - z_j)$ be a polynomial of degree n having all its zeros in $|z| \leqslant k$, $k \geqslant 1$, then Aziz [Proc. Am. Math. Soc., **89**, (1983) 259–266] proved

$$\max_{|z|=1} |P'(z)| \geqslant \frac{2}{1+k^n} \sum_{j=1}^{n} \frac{k}{k+|z_j|} \max_{|z|=1} |P(z)|.$$

In this paper, we prove a polar derivative extension which sharpens the above inequality. As a consequence, we also derive a result on Bernstein type inequality for the class of polynomials having all its zeros in $|z| \geqslant k$, $k \leqslant 1$.

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