SUMUDU TRANSFORM AND THE STABILITY OF
SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we introduce a new integral transform, namely, Sumudu transform and we apply the transform to investigate the Hyers-Ulam stability, Hyers-Ulam-Rassias stability, Mittag-Leffler-Hyers-Ulam stability and Mittag-Leffler-Hyers-Ulam-Rassias stability of second order linear differential equations.

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