

GLOBAL REGULARITY IN PARABOLIC WEIGHTED ORLICZ–MORREY SPACES OF SOLUTIONS TO PARABOLIC EQUATIONS WITH *VMO* COEFFICIENTS

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Abstract. We show continuity in parabolic generalized weighted Orlicz–Morrey spaces $M_w^{\Phi, \varphi}$ of sublinear integral operators generated by parabolic singular and nonsingular operators and their commutators with *BMO* functions. The obtained estimates are used to study global regularity of the solution of the Cauchy–Dirichlet problem for linear uniformly parabolic operators of second order with discontinuous data.

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