Journal of Mathematical Inequalities
Volume 18, Number 3 (2024), 895–910 doi:10.7153/jmi-2024-18-48

UPPER BOUNDS ON THE HARMONIC STATUS INDEX

MAHDIEH AZARI

Abstract. The harmonic status index of a simple connected graph G is defined as the sum of the weights $\frac{2}{\sigma_G(u)+\sigma_G(v)}$ over all edges uv of G, where $\sigma_G(u)$ denotes the status of the vertex u in G which is the sum of distances between u and all other vertices of G. In this paper, we present upper bounds on the harmonic status index of some families of graph products in terms of certain structural invariants such as the order, size, maximum degree, inverse status and harmonic status index of their components. The graph products considered here are sum, disjunction, symmetric difference, Indu-Bala product, corona product, Cartesian product, lexicographic product, and strong product. Some applications of the obtained results are also presented as corollaries.

Mathematics subject classification (2020): 05C12, 05C76, 05C35, 92E10.
Keywords and phrases: Status of a vertex, graph invariant, graph product, upper bound.

REFERENCES

[1] A. ALI, *Counter examples to a conjecture concerning harmonic index*, Asian-Eur. J. Math., **11**, 03 (2018), 1850035.
[2] A. ALI, S. BALACHANDRAN, S. ELUMALAI AND T. MANSOUR, *On the n-vertex trees with sixth to fifteenth maximum harmonic indices*, Afr. Mat., **31**, 5–6 (2020), 771–780.
[3] A. ALI, L. ZHONG AND I. GUTMAN, *Harmonic index and its generalizations: Extremal results and bounds*, MATCH Commun. Math. Comput. Chem., **81**, 02 (2019), 249–311.
[4] M. AZARI, *Graph products and eccentric harmonic index*, Asian-Eur. J. Math., **15**, 2 (2022), 2250027.
[5] M. AZARI AND A. IRANMANESH, *Computing the eccentric-distance sum for graph operations*, Discrete Appl. Math., **161**, 18 (2013), 2827–2840.
[6] M. AZARI AND A. IRANMANESH, *Some inequalities for the multiplicative sum Zagreb index of graph operations*, J. Math. Inequal., **9**, 3 (2015), 727–738.
[7] Y. M. CHU, M. K. SHAFIQ, M. IMRAN, M. K. SIDDIQUI, H. M. A. SIDDIQUI, S. BABY, M. CANCAN, *The sharp bounds of Zagreb indices on connected graphs*, J. Math. Inequal., **15**, 2 (2021), 477–489.
[8] T. DOŠLIĆ, *Vertex-Weighted Wiener polynomials for composite graphs*, Ars Math. Contemp., **1**, 1 (2008), 66–80.
[9] T. DOŠLIĆ AND M. SAHELI, *Eccentric connectivity index of composite graphs*, Util. Math., **95**, (2014), 3–22.
[10] S. FAJTLOWICZ, *On conjectures of Graffiti-II*, Congr. Numer., **60**, (1987), 187–197.
[11] J. C. HERNÁNDEZ-GÓMEZ, J. A. MÉNDEZ-BERMÚDEZ, J. M. RODRÍGUEZ AND J. M. SIGARRETA, *Harmonic index and harmonic polynomial on graph operations*, Symmetry, **10**, 10 (2018), 456.
[12] G. INDULAL AND R. BALAKRISHNAN, *Distance spectrum of Indu-Bala product of graphs*, AKCE Int. J. Graphs Comb., **13**, 3 (2016), 230–234.
[13] S. R. JOG AND S. L. PATIL, *On status indices of some graphs*, Int. J. Math. Combin., **3**, (2018), 99–107.
[14] M. H. KHALIFEH, H. YOUSEFI-AZARI AND A. R. ASHRAFI, *The first and second Zagreb indices of some graph operations*, Discrete Appl. Math., **157**, 4 (2009), 804–811.
[15] Y. NACAROGLU AND A. D. MADEN, *The upper bounds for multiplicative sum Zagreb index of some graph operations*, J. Math. Inequal., **11**, 3 (2017), 749–761.
[16] C. NICULESCU AND L. E. PERSSON, *Convex functions and their applications: a contemporary approach*, Springer, New York, 2006.

Paper JMI-18-48

[17] B. N. ONAGH, *The harmonic index of product graphs*, Math. Sci., **11**, 3 (2017), 203–209.

[18] K. PATTABIRAMAN AND A. SANTHAKUMAR, *Status connectivity indices and its coindices of composite graphs*, Int. J. Math. Combin., **2**, (2019), 104–113.

[19] H. S. RAMANE, B. BASAVANAGOUD AND A. S. YALNAIK, *Harmonic status index of graphs*, Bull. Math. Sci. Appl., **17**, (2016), 24–32.

[20] B. SHWETHA SHETTY, V. LOKESHA AND R. S. RANJINI, *On the harmonic index of graph operations*, Trans. Comb., **4**, 4 (2015), 5–14.

[21] X. SUN, Y. GAO AND J. DU, *The harmonic index of two-trees and quasi-tree graphs*, J. Math. Inequal., **13**, 2 (2019), 479–493.

[22] M. TAVAKOLI, F. RAHBARNIA AND A. R. ASHRAFI, *Note on strong product of graphs*, Kragujevac J. Math., **37**, 1 (2013), 187–193.