

SOME SINGULAR VALUE INEQUALITIES FOR MATRICES

AHMAD AL-NATOOR, ALIAA BURQAN*, MOHAMMAD A. AMLEH
AND CRISTIAN CONDE

Abstract. In this paper, we prove some singular value inequalities for sums and products of matrices. Some of our inequalities will give several generalizations of recent known inequalities. Among other inequalities, we prove that if A, B, C, D, X, Y are $n \times n$ complex matrices such that X and Y are positive semidefinite, then

$$s_j(AXB^* + CYD^*) \leq \sqrt{\left(\|A\|^2 + \|C\|^2 \right) \left(\|B\|^2 + \|D\|^2 \right)} s_j(X \oplus Y),$$

for $j = 1, 2, \dots, n$, which is a generalization of an inequality in [12]. Here, s_j and $\|\cdot\|$ denote the singular value and the spectral norm of matrices, respectively.

Mathematics subject classification (2020): 15A18, 15A42, 15A45, 15A60, 15B57, 47B30.

Keywords and phrases: Positive semidefinite matrix, singular value, inequality.

REFERENCES

- [1] H. ALBADAWI, *Singular values and arithmetic-geometric mean inequalities for operators*, Ann. Funct. Anal. **3**, 10–18 (2012).
- [2] A. AL-NATOOR, M. A. AMLEH, B. ABUGHAZALLEH, A. BURQAN, *Generalization of some unitarily invariant norm inequalities for matrices*, J. Math. Inequal. **17** (2), 581–589 (2023).
- [3] A. AL-NATOOR, S. BENZAMIA, F. KITTANEH, *Unitarily invariant norm inequalities for positive semidefinite matrices*, Linear Algebra Appl. **633**, 303–315 (2022).
- [4] A. AL-NATOOR, O. HIRZALLAH, F. KITTANEH, *Interpolating inequalities for functions of positive semidefinite matrices*, Banach J. Math. Anal. **12**, 955–969 (2018).
- [5] A. AL-NATOOR, O. HIRZALLAH, F. KITTANEH, *Singular value and norm inequalities involving the numerical radii of matrices*, Ann. Funct. Anal. **15** (2024), Paper No. 7.
- [6] A. AL-NATOOR, O. HIRZALLAH, F. KITTANEH, *Singular value inequalities for convex functions of positive semidefinite matrices*, Ann. Funct. Anal. **17** (2023), Paper No. 7.
- [7] A. AL-NATOOR, O. HIRZALLAH, F. KITTANEH, *Singular value and norm inequalities for product and sums of matrices*, Period. Math. Hung. **88**, 204–217 (2024).
- [8] A. AL-NATOOR, F. KITTANEH, *Further unitarily invariant norm inequalities for positive semidefinite matrices*, Positivity **26**, 11 (2022), Paper No. 8.
- [9] A. AL-NATOOR, F. KITTANEH, *Singular value and norm inequalities for positive semidefinite matrices*, Linear Multilinear Algebra **70**, 4498–4507 (2022).
- [10] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York (1997).
- [11] A. BURQAN, F. KITTANEH, *Singular value and norm Inequalities associated with 2×2 positive semidefinite block matrices*, The Electronic Journal of Linear Algebra **32**, 116–124 (2017).
- [12] O. HIRZALLAH, F. KITTANEH, *Inequalities for sums and direct sums of Hilbert space operator*, Linear Algebra Appl. **424**, 71–82 (2007).
- [13] H. R. MORADI, W. AUDEH, M. SABABHEH, *Singular values inequalities via matrix monotone functions*, Anal. Math. Phys. **13** (71) (2023), <https://doi.org/10.1007/s13324-023-00832-8>.
- [14] M. SABABHEH, S. FURUICHI, S. SHEYBANI, H. R. MORADI, *Singular values inequalities for matrix means*, J. Math. Inequal. **16** (1), 169–179 (2022).

- [15] X. ZHAN, *Singular values of differences of positive semidefinite matrices*, SIAM J. Matrix Anal. Appl. **22**, 819–823 (2002).
- [16] L. ZOU, *An arithmetic-geometric mean inequality for singular values and its applications*, Linear Algebra Appl. **528**, 25–32 (2017).