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REVERSED HERMITE–HADAMARD INEQUALITY WITH APPLICATIONS

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Abstract. The Hermite-Hadamard inequality is one of the most interesting inequalities that give lower and upper bounds of the mean value of a convex function in a way that refines the convex characteristic of the function.

This paper presents a new reversed version of this outstanding result, with applications toward means of positive numbers, operator inequalities, and the Riemann-Liouville fractional integrals.

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