

## NOVEL BOUNDS FOR THE EUCLIDEAN OPERATOR RADIUS OF HILBERT SPACE OPERATOR PAIRS

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*Abstract.* This paper aims to establish new upper bounds for the Euclidean operator radius concerning pairs of bounded linear operators in a complex Hilbert space. To achieve this objective, we utilize some Boas-Bellman type inequalities as proof tools. Furthermore, we extend our findings to derive novel upper bounds for the numerical radius of operators in Hilbert spaces. These results contribute to advancing our understanding and analytical capabilities regarding operator properties within the framework of Hilbert spaces.

*Mathematics subject classification (2020):* 47A63, 47A12, 47A05, 47A30.

*Keywords and phrases:* Numerical radius, pair of operators, Euclidean operator radius, Boas-Bellman inequality.

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