

MULTIPLE-TERM IMPROVEMENTS OF JENSEN'S INEQUALITY FOR (p, h) -CONVEX AND (p, h) -LOG CONVEX FUNCTIONS

DUONG QUOC HUY, ABDELMAJID GOURTY, MOHAMED AMINE IGHACHANE*
AND MOHAMED BOUMAZGOUR

Abstract. In this paper, we present several new multiple-term improvements of Jensen's inequality for (p, h) -convex and (p, h) -log convex functions. As applications of our results, we present new bounds by employing means and Hölder type inequalities for the symmetric norms for τ -measurable operators. We make links between our findings and a number of well-known discoveries in the literature.

Mathematics subject classification (2020): 26D07.

Keywords and phrases: (p, h) -convex function, (p, h) -log-convex functions, weak sub-majorization, Jensen's inequality, scalar means.

REFERENCES

- [1] D. CHOI, M. KRNIĆ AND J. PEČARIĆ, *Improved Jensen-type inequalities via linear interpolation and applications*, J. Math. Anal. Appl. **11** (2) (2017), 301–322.
- [2] Y. DENG, H. ULLAH, M. A. KHAN, S. IQBAL, AND S. WU, *Refinements of Jensen's inequality via majorization results with applications in the information theory*, J. Math. (2021), <https://doi.org/10.1155/2021/1951799>.
- [3] X. T. DINH, H. Q. DUONG AND H. N. NGUYEN, *Two new extensions of the weighted arithmetic-geometric mean inequality via weak sub-majorization*, Indian. J. Pure. Appl. Math. (2022), <https://doi.org/10.1007/s13226-022-00223-y>.
- [4] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional*, Bull. Austral. Math. Soc. **74** (3) (2006), 471–478.
- [5] B. FUGLEDE AND R. V. KADISON, *On determinants and a property of the trace in finite factors*, Proc. Nat. Acad. Sci. U.S.A. **37** (1951), 425–431.
- [6] B. FUGLEDE, R. V. KADISON, *Determinant theory in finite factors*, Ann. of Math. **55** (1952), no. 2, 520–530.
- [7] Y. HAN, *Some determinant inequalities for operators*, Linear. Multilinear. Algebra. **67** (1) (2019), 94–105.
- [8] M. A. IGHACHANE AND M. BOUCHANGOUR, *New inequalities for (p, h) -convex functions for τ -measurable operators*, Filomat. **37** (16) (2023), 5259–5271.
- [9] M. A. IGHACHANE AND M. BOUCHANGOUR, *Improved Jensen's type inequality for (p, h) -convex functions via weak sub-majorization*, Filomat. **38** (5) (2024), 1793–1806.
- [10] M. A. IGHACHANE AND M. BOUCHANGOUR, *Some refinements of real power form inequalities for convex functions via weak sub-majorization*, Oper. Matrices. **17** (1) (2023), 213–233.
- [11] M. A. IGHACHANE, M. BOUCHANGOUR AND Z. TAKI, *Some refinements of real power form inequalities for (p, h) -convex functions via weak sub-majorization*, Oper. Matrices. **17** (3) (2023), 793–808.
- [12] M. A. IGHACHANE, D. Q. HUY, DOAN T. T. VAN AND M. BOUCHANGOUR, *Further refinements of real power form inequalities for convex functions via weak sub-majorization*, Rend. Circ. Mat. Palermo, II. Ser (2023), <https://doi.org/10.1007/s12215-023-00974-5>.
- [13] M. A. IGHACHANE, L. SADEK AND M. SABABHEH, *Improved Jensen's type inequalities for (p, h) convex functions with applications*, Kragujevac. J. of Math. **50** (1) (2026), 71–89.

- [14] A. W. MARSHALL, I. OLKIN AND B. C. ARNOLD, *Inequalities: Theory of Majorization and Its Applications*, second edition, Springer Series in Statistics, Springer, New York (2011).
- [15] J. SHAO, *Two variables functionals and inequalities related to measurable operators*, J. Inequ. Appl. **304**, (2017), <https://doi.org/10.1186/s13660-017-1583-9>.
- [16] M. SABABHEH, *Improved Jensen's inequality*, J. Inequal. Appl. **20** (2) (2017), 389–403.
- [17] M. SABABHEH, *Convexity and matrix means*, Linear Algebra App. **506** (2016), 588–602.
- [18] M. SABABHEH, *Log and harmonically log-convex functions related to matrix norms*, Oper. Matrices. **10** (2) (2016), 453–465.
- [19] M. SABABHEH, *Graph indices via the AM-GM inequality*, Discrete. Appl. Math. **230** (4) (2017), 100–111.
- [20] M. SABABHEH, *Means refinements via convexity*, Mediterr. J. Math. **14**, 125 (2017), <https://doi.org/10.1007/s00009-017-0924-8>.
- [21] M. SABABHEH, *Convex functions and means of matrices*, Math. Ineq. Appl. **20** (1) (2017), 29–47.
- [22] M. SABABHEH, H. R. MORADI AND S. FURUICHI, *Operator inequalities via geometric convexity*, Math. Inequal. Appl. **22** (4) (2019), 1215–1231.
- [23] T. SAEED, M. A. KHAN AND H. ULLAH, *Refinements of Jensen's inequality and applications*, AIMS. Mathematics. **7** (4) (2022), 5328–5346, doi:10.3934/math.2022297.
- [24] G. PISIER AND Q. XU, *Non-commutative L_p spaces*, Handbook of the geometry of Banach spaces, vol. **2**, North-Holland, Amsterdam, 2003.
- [25] K. S. ZHANG AND J. P. WAN, *p -convex functions and their properties*, Pure. Appl. Math. **23** (1) (2007), 130–133.
- [26] J. ZHOU, Y. WANG AND T. WU, *A Schwarz inequality for τ -measurable operators A^*XB^** , J. Xinjiang. Univ. Naatur. Sci. **1** (2009), 69–73.