

SOME GENERALIZED INEQUALITIES FOR ACCRETIVE–DISSIPATIVE MATRICES

YONGHUI REN

Abstract. In this paper, we present some generalized inequalities for accretive-dissipative matrices involving convex and concave functions which extend some results of Jabbarzadeh and Kaleibary. Among other results, we show that if $T_1, T_2, \dots, T_n \in \mathbb{M}_n(\mathbb{C})$ are accretive-dissipative matrices, then for every non-negative increasing concave function f on $[0, \infty)$ and $p \geq 1$, we have

$$\left\| f\left(\sqrt{2} \left| \sum_{j=1}^n T_j \right| \right) \right\|_p^p \leq 2 \cdot n^{p-1} \sum_{j=1}^n \left\| f(|T_j|) \right\|_p^p.$$

Moreover, we also provide the generalized forms of Minkowski's determinant inequality and the Young type determinant inequality involving accretive-dissipative matrices.

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REFERENCES

- [1] R. BHATIA, *Matrix Analysis*, vol. 169, Springer-Verlag, 1997.
- [2] R. BHATIA, F. KITTANEH, *Cartesian decompositions and Schatten norms*, *Linear Algebra Appl.*, **318** (2000) 109–116.
- [3] J. C. BOURIN, E. Y. LEE, *Unitary orbits of Hermitian operators with convex or concave functions*, *Bull. London Math. Soc.*, **44** (2012) 1085–1102.
- [4] J. C. BOURIN, M. UCHIYAMA, *A matrix subadditivity inequality for $f(A+B)$ and $f(A)+f(B)$* , *Linear Algebra Appl.*, **423** (2007) 512–518.
- [5] F. HIAI, D. PETZ, *Introduction to Matrix Analysis and Applications*, Springer International Publishing, 2014.
- [6] R. A. HORN, C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, Cambridge, 2013.
- [7] M. R. JABBARZADEH, V. KALEIBARY, *Inequalities for accretive-dissipative block matrices involving convex and concave functions*, *Linear Multilinear Algebra* **70** (2022), 395–410.
- [8] F. KITTANEH, M. SAKKIJHA, *Inequalities for accretive-dissipative matrices*, *Linear Multilinear Algebra* **67** (2019) 1037–1042.
- [9] T. KOSEM, *Inequalities between $\|f(A+B)\|$ and $\|f(A)+f(B)\|$* , *Linear Algebra Appl.*, **418** (2006) 153–160.
- [10] M. LIN, *Fisher type determinantal inequalities for accretive-dissipative matrices*, *Linear Algebra Appl.*, **438** (2013) 2808–2812.
- [11] M. LIN, *Extension of a result of Hanynsworth and Hartfiel*, *Arch. Math.*, **104** (2015) 93–100.
- [12] Y. REN, C. YANG, *Some generalizations of numerical radii and Schatten p -norms inequalities*, *J. Math. Inequal.*, **17** (2023) 1371–1386.
- [13] M. UCHIYAMA, *Subadditivity of eigenvalue sums*, *Proc. Amer. Math. Soc.*, **134** (2006) 1405–1412.
- [14] C. YANG, F. LU, *Some generalizations of inequalities for sector matrices*, *J. Inequal. Appl.*, **2018**, Paper No. 183, 11 pp.
- [15] F. ZHANG, *A matrix decomposition and its applications*, *Linear Multilinear Algebra* **63** (2015) 2033–2042.