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ON THE CONVERGENCE PROPERTIES OF DURRMEYER TYPE EXPONENTIAL SAMPLING SERIES IN (MELLIN) ORLICZ SPACES

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Abstract. In this study, by using the concept of modular convergence with the help of a suitable modular functional we obtain main theorem for the (Mellin) Orlicz spaces $X_0^{\eta} = L_{\mu}^{\phi}(\mathbb{R}^+)$ whose functions don't have to be bounded or continuous. Then we customize our theorems for $L_{\mu}^{p}(\mathbb{R}^+)$-space and $L_{\mu}^{\eta_{\alpha,\beta}}(\mathbb{R}^+)$ using these results. Finally, examples with graphical representations are given for some Durrmeyer type exponential sampling series with special kernels.

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