Journal of Mathematical Inequalities
Volume 18, Number 3 (2024), 1171–1185
doi:10.7153/jmi-2024-18-66

EXTENSIONS OF MATRIX MEAN INEQUALITIES TO SECTOR MATRICES

LEILA NASIRI* AND SHIGERU FURUICHI

Abstract. In this paper, extesions of some inequalities involving matrix means and sector matrices are considered. Among other results, we prove that if two sector matrices A and B satisfy $0 < mI_n \leqslant \Re(A), \Re(B) \leqslant MI_n$, then

$$\Phi^p(\Re(A\sigma B)) \leqslant \sec^{2p}(\theta) K^p(h) \Phi^p(\Re(B\sigma^{\perp} A)), \quad (0 \leqslant p \leqslant 2)$$

for every unital positive linear map Φ and arbitrary mean σ, where $K(h) := \frac{(M+m)^2}{4Mm}$ is the Kantorovich constant with $h := \frac{M}{m}$. In addition, we present some norm, numerical radius and determinantal inequalities for sector matrices.

Mathematics subject classification (2020): 47A63, 47A64.
Keywords and phrases: Matrix mean, sector marix, positive linear map, Kantorovich constant, unitaliry invariant norm.

REFERENCES

[1] D. AFRAZ, R. LASHKARIPOUR AND M. BAKHERAD, *Norm inequalities involving a special class of functions for sector matrices*, J. Inequal. Appl. **2020** (2020), 122.
[2] T. ANDO AND X. ZHAN, *Norm inequalities related to operator monoton functions*, Math. Ann. **315** (1999), 771–780.
[3] T. ANDO, *Majorization and inequalities in matrix theory*, Linear Algebra Appl. **199** (1994), 17–67.
[4] T. ANDO, *Concavity of certain maps on positive definite matrices and applications to Hadamard prodcts*, Linear Algebra Appl. **26** (1979), 203–241.
[5] M. ALOMARI, S. SAHOO AND M. BAKHERAD, *Further numerical radius inequalities*, J. Math. Inequal. **16** (1) (2022), 307–326.
[6] M. BAKHERAD, *Refinements of a reversed AM-GM operator inequality*, Linear Multilinear Algebra **64** (2016), no. 9, 1687–1695.
[7] M. BAKHERAD, M. HAJMOHAMADI, R. LASHKARIPOUR AND M. SABABHEH, *Some estimates for the generalized numerical radius*, J. Anal. **31** (3) (2023), 2163–2172.
[8] Y. BEDRANI, F. KITTANEH AND M. SABABHEH, *From positive to accretive matrices*, Positivity **25** (2021), 1601–1629.
[9] Y. BEDRANI, F. KITTANEH AND M. SABABHEH, *Numerical radii of accretive matrices*, Linear and multilinear Algebra **69** (2021), 957–970.
[10] R. BHATIA AND F. KITTANEH, *Notes on matrix arithmetic-geometric mean inequalities*, Linear Algebra Appl. **308** (2000), no. 1–3, 203–211.
[11] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York 1997.
[12] P. BHUNIA, S. S. DRAGOMIR, M. S. MOSLEHIAN AND K. PAUL, *Lectures on Numerical Radius Inequalities*, Infosys Science Foundation Series in Mathematical Sciences, Springer–Verlag, 2022.
[13] I. H. GÜMÜŞ, H. R. MORADI AND M. SABABHEH, *More accurate operator means inequalities*, J. Math. Anal. Appl. **465** (1) (2018), 267–280.
[14] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, Cambridge, 2013.
[15] F. KUBO AND T. ANDO, *Means of positive linear operators*, Math. Ann. **248** (1980), 205–224.
[16] M. LIN, *Extension of a result of Hanynsworth and Hartfiel*, Arch. Math. **104** (2015), 93–100.

Paper JMI-18-66

[17] L. NASIRI AND M. BAKHERAD, *New generalized inequalities using arbitrary operator means and their duals*, An. Univ. Craiova Ser. Mat. Inform. **48** (1) (2021), 1–9.
[18] L. NASIRI AND S. FURUICHI, *On a reverse of the Tan-Xie inequality for sector matrices and its applications*, J. Math. Inequal. **15** (2021), 1425–1434.
[19] J. E. PEČARIĆ, T. FURUTA, J. MIĆIĆ HOT AND Y. SEO, *Mond-Pečarić Method in operator inequalities*, Element, Zagreb, 2005.
[20] C. YANG AND F. LU, *Some generalizations of inequalities for sector matrices*, J. Inequal. Appl. (2018), Art. 183.
[21] F. ZHANG, *A matrix decomposition and its applications*, Linear Multilinear Algebra **63** (2015), 2033–2042.
[22] D. ZHANG AND N. ZHANG, *Some norm inequalities for upper sector matrices*, Filomat **35** (5) (2021).