

## BALL-COVERING OF PRODUCT SPACES AND GÂTEAUX DIFFERENTIABILITY OF THE CENTER

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*Abstract.* In this paper, the author proves that if  $X_1, X_2$  are Banach spaces, there exists a real number  $\alpha > 0$  and a ball covering  $\mathcal{B}_i$  of  $X_i$  such that  $\mathcal{B}_i$  is  $\alpha$ -off the origin and the ball-covering point is a norm Gâteaux differentiability point if and only if there exists a real number  $\alpha > 0$  and a ball covering  $\mathcal{B}$  of  $(X_1 \times X_2, \|\cdot\|_\infty), (X_1 \times X_2, \|\cdot\|_p)$  such that  $\mathcal{B}$  is  $\alpha$ -off the origin and the ball-covering point is a norm Gâteaux differentiability point.

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