Journal of Mathematical Inequalities
Volume 18, Number 4 (2024), 1233–1245
doi:10.7153/jmi-2024-18-70

THE MINIMAL SYSTEM OF GENERATORS OF AN AFFINE, PLANE AND NORMAL SEMIGROUP

M. A. MORENO-FRÍAS* AND J. C. ROSALES

Abstract. If X is a nonempty subset of $\mathbb{Q}^k$, the *cone* generated by X is $\mathrm{C}(X) = \{q_1x_1 + \cdots + q_nx_n \mid n \in \mathbb{N} \setminus \{0\}, \{q_1,\ldots,q_n\} \subseteq \mathbb{Q}_0^+$ and $\{x_1,\ldots,x_n\} \subseteq X\}$. In this work we present an algorithm which calculates from $\{(a_1,b_1),(a_2,b_2)\} \subseteq \mathbb{N}^2$, the minimal system of generators of the affine semigroup $\mathrm{C}(\{(a_1,b_1),(a_2,b_2)\}) \cap \mathbb{N}^2$. This algorithm is based on the study of proportionally modular Diophantine inequalities carried out in [1]. Also, we present an upper bound for the embedding dimension of this semigroup.

Mathematics subject classification (2020): 20M14, 32B25.
Keywords and phrases: Affine semigroup, Bézout sequence, normal semigroup, plane semigroup, triangulation, embedding dimension.

REFERENCES

[1] M. BULLEJOS AND J. C. ROSALES, *Proportionally modular Diophantine inequalities and the Stern-Brocot tree*, Mathematics of Computation **78** (2009), 1211–1226.
[2] R. E. GRAHAM, D. E. KNUTH AND O. PATASHNIK, *Concrete Mathematics. A foundation for Computer Science*, Second ed. Addison-Wesley, Reading, MA, 1994.
[3] M. HOCHSTER, *Rings of invariants of tori, Cohen-Macaulay rings generated by monomials, and polytopes*, Ann. Math. **96** (1972), 318–337.
[4] G. LACHAUD, *Klein polygons and geometric diagrams*, Contemporary Mathematics, **210** (1998), 365–372.
[5] J. C. ROSALES AND P. A. GARCÍA-SÁNCHEZ, *Finitely Generated Commutative Monoids*, Nova Science, New York, 1999.
[6] J. C. ROSALES AND P. A. GARCÍA-SÁNCHEZ, *On normal affine semigroups*, Linear Algebra Appl. **286** (1999), 175–186.
[7] J. C. ROSALES, P. A. GARCÍA-SÁNCHEZ AND J. M. URBANO-BLANCO, *The set of solutions of a proportionally modular Diophantine inequality*, J. Number Theory **128** (2008), 453–467.

Paper JMI-18-70