OPERATOR QUADRATIC MEAN AND POSITIVE LINEAR MAPS

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Abstract. This paper presents several bounds for unital positive linear mappings and the so-called quadratic mean. Some of these results can be viewed as sub-multiplicative inequalities, while others are Cauchy-Schwarz-type inequalities. Related results that treat the tensor products will be presented, too. Dragomir's result is improved by using the inequalities by Fujii–Nakamoto. As applications, we present some numerical radius inequalities by the obtained results. Finally, we study Ando-type inequalities for the quadratic mean.

Mathematics subject classification (2020): Primary 47A63, 47A64; Secondary 47A30, 47A12, 15A60.
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REFERENCES

[1] T. ANDO, *Concavity of certain maps on positive definite matrices and applications to Hadamard products*, Linear Algebra Appl., **26** (1979), 203–241.

[2] R. BHATIA, R. SHARMA, *Some inequalities for positive linear maps*, Linear Algebra Appl., **436** (2012), 1562–1571.

[3] R. BHATIA, *Matrix Analysis*, Graduate Texts in Mathematics., Springer, New York (1997).

[4] R. BHATIA, *Positive Definite Matrices*, Princeton University Press, 2007.

[5] M. D. CHOI, *Some assorted inequalities for positive linear maps on C*-algebras*, J. Operator Theory., **4** (1980), 271–285.

[6] M. D. CHOI, *A Schwarz inequality for positive linear maps on C*-algebras*, Illinois J. Math., **18** (1974), 565–574.

[7] C. DAVIS, *A Schwarz inequality for convex operator functions*, Proc. Amer. Math. Soc., **8** (1957), 42–44.

[8] S. S. DRAGOMIR, *Some inequalities of Hölder type for the quadratic weighted geometric mean of bounded linear operators in Hilbert spaces*, Linear Multilinear Algebra., **66** (2) (2018), 268–279.

[9] J. I. FUJII, *Operator-valued inner product and operator inequalities*, Banach J. Math. Anal., **2** (2) (2008), 59–67.

[10] M. FUJII, R. NAKAMOTO, *Refinements of Hölder–McCarthy inequality and Young inequality*, Adv. Oper. Theory., **1** (2) (2016), 184–188.

[11] M. FUJIMOTO, Y. SEO, *The Schwarz inequality via operator-valued inner product and the geometric operator mean*, Linear Algebra Appl., **561** (2019), 141–160.

[12] S. FURUICHI, H. R. MORADI, *Advances in Mathematical Inequalities*, De Gruyter, 2020.

[13] S. FURUICHI, K. YANAGI, AND H. R. MORADI, *Mathematical inequalities on some weighted means*, J. Math. Inequal., **17** (2) (2023), 447–457.

[14] R. V. KADISON, *A generalized Schwarz inequality and algebraic invariants for operator algebras*, Ann. Math., **56** (1952), 494–503.

[15] F. KUBO, T. ANDO, *Means of positive linear operators*, Math. Ann., **246** (1980), 205–224.

[16] C. K. LI, R. MATHIAS, *Matrix inequalities involving a positive linear map*, Linear Multilinear Algebra., **41** (1996), 221–231.

[17] R. MATHIAS, *A note on: "More operator versions of the Schwarz inequality" [Comm. Math. Phys. 215 (2000), no. 2, 239–244; by R. Bhatia and C. Davis]*, Positivity., **8** (2004), 85–87.

[18] J. MIĆIĆ, J. PEČARIĆ, AND Y. SEO, *Complementary inequalities to inequalities of Jensen and Ando based on the Mond-Pečarić method*, Linear Algebra Appl., **318** (2000), 87–108.

[19] J. MIĆIĆ, J. PEČARIĆ, Y. SEO, AND M. TOMINAGA, *Inequalities for positive linear maps on Hermitian matrices*, Math. Inequal. Appl., **3** (2000), 559–591.

[20] B. MOND, J. E. PEČARIĆ, *On matrix convexity of the Moore–Penrose inverse*, Internat. J. Math. & Math. Sci., **19** (4) (1996), 707–710.

[21] H. R. MORADI, I. H. GÜMÜŞ, AND Z, HEYDARBEYGI, *A glimpse at the operator Kantorovich inequality*, Linear Multilinear Algebra., **67** (5) (2019), 1031–1036.

[22] H. R. MORADI, M. E. OMIDVAR, I. H. GÜMÜŞ, AND R. NASERI, *A note on some inequalities for positive linear maps*, Linear Multilinear Algebra., **66** (7) (2018), 1449–1460.

[23] R. NAKAYAMA, Y. SEO, AND R. TOJO, *Matrix Hölder-McCarthy inequality via matrix geometric means*, Adv. Oper. Theory., **5** (2020), 744–767.

[24] R. PAL, M. SINGH, M. S. MOSLEHIAN, AND J. S. AUJLA, *A new class of operator monotone functions via operator means*, Linear Multilinear Algebra., **64** (12) (2016), 2463–247.

[25] M. SABABHEH, *Numerical radius inequalities via convexity*, Linear Algebra Appl., **549** (2018), 67–78.

[26] M. SABABHEH, H. R. MORADI, AND S. FURUICHI, *Exponential inequalities for positive linear mappings*, J. Funct. Spaces., (2018), Article ID 5467413.

[27] A. SHEIKHHOSSEINI, *An arithmetic-geometric mean inequality related to numerical radius of matrices*, Konuralp J. Math., **5** (2017), 85–91.

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