

INEQUALITIES INVOLVING BEREZIN NORM AND BEREZIN NUMBER OF HILBERT SPACE OPERATORS

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Abstract. This paper presents several Berezin number and norm inequalities for Hilbert space operators. These inequalities improve some earlier related inequalities. Among other inequalities, it is shown that if A is a bounded linear operator on a Hilbert space, then

$$\mathbf{ber}^2(A) \leq \left\| \frac{A^*A + AA^*}{2} - \frac{1}{2R} \left((1-t)A^*A + tAA^* - \left((1-t)(A^*A)^{\frac{1}{2}} + t(AA^*)^{\frac{1}{2}} \right)^2 \right) \right\|_{\mathbf{ber}}$$

where $R = \max\{t, 1-t\}$ and $0 \leq t \leq 1$.

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REFERENCES

- [1] M. BAKHERAD, *Some Berezin number inequalities for operator matrices*, Czechoslovak Math. J., **68** (2018), 997–1009.
- [2] M. BAKHERAD, M. HAJMOHAMADI, R. LASHKARIPOUR, AND S. SAHOO, *Some extensions of Berezin number inequalities on operators*, Rocky Mt. J. Math., **51**(6) (2021), 1941–1951.
- [3] F. A. BEREZIN, *Covariant and contravariant symbols for operators*, Math USSR-Izv., **6** (1972), 1117–1151.
- [4] F. A. BEREZIN, *Quantizations*, Math USSR-Izv., **8** (1974), 1109–1163.
- [5] R. BHATIA, *Positive Definite Matrices*, Princeton University Press, Princeton, (2007).
- [6] P. BHUNIA, K. PAUL, AND A. SEN, *Inequalities involving Berezin norm and Berezin number*, Complex Anal. Oper. Theory, **17**, 7 (2023), <https://doi.org/10.1007/s11785-022-01305-9>.
- [7] M. L. BUZANO, *Generalizzazione della diseguaglianza di Cauchy-Schwarz*, Rend. Sem. Mat. Univ. Politech. Torino, **31** (1971/73), 405–409 (1974) (in Italian).
- [8] M. EL-HADDAD, F. KITTANEH, *Numerical radius inequalities for Hilbert space operators. II*, Studia Math., **182** (2007), 133–140.
- [9] S. FURUICHI, H. R. MORADI, AND M. SABABHEH, *New inequalities for interpolational operator means*, J. Math. Inequal., **15** (1) (2021), 107–116.
- [10] M. GUESBA, M. SABABHEH, *On the Berezin number of operator matrices*, Oper. Matrices., **18** (1) (2024), 129–146.
- [11] M. GUESBA, M. T. GARAYEV, *Estimates for the Berezin number inequalities*, J. Pseudo-Differ. Oper. Appl., (2024) **15**: 43, <https://doi.org/10.1007/s11868-024-00612-3>.
- [12] M. GUESBA, *Some Euclidean Berezin number inequalities of a pair of operators and their applications*, Filomat., **37** (26) (2023).
- [13] M. GUESBA, F. KITTANEH, *Berezin number inequalities for sums and products of operators and applications*, Lobachevskii J. Math., **45** (3) (2024), 1055–1065.
- [14] P. R. HALMOS, *A Hilbert Space Problem Book*, 2nd ed, New York: Springer, 1982.
- [15] O. HIRZALLAH, F. KITTANEH, AND K. SHEBRAWI, *Numerical radius inequalities for certain 2×2 operator matrices*, Integr. Equ. Oper. Theory., **71** (2011), 129–147.
- [16] M. T. KARA EV, *Berezin symbol and invertibility of operators on the functional Hilbert spaces*, J. Funct Anal., **238** (2006), 181–192.

- [17] F. KITTANEH, H. R. MORADI, *Cauchy-Schwarz type inequalities and applications to numerical radius inequalities*, *Math. Inequal. Appl.*, **23** (3) (2020), 1117–1125.
- [18] F. KITTANEH, *Norm inequalities for sums of positive operators*, *J. Operator Theory.*, **48** (2002), 95–103.
- [19] F. KITTANEH, *Notes on some inequalities for Hilbert Space operators*, *Publ. Res. Inst. Math. Sci.*, **24** (1988), 283–293.
- [20] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, *Studia Math.*, **168** (2005), 73–80.
- [21] H. R. MORADI, M. SABABHEH, *More accurate numerical radius inequalities (II)*, *Linear Multilinear Algebra.*, **69** (5) (2021), 921–933.
- [22] M. E. OMIÐVAR, H. R. MORADI, *Better bounds on the numerical radii of Hilbert space operators*, *Linear Algebra Appl.*, **604** (2020), 265–277.
- [23] M. E. OMIÐVAR, H. R. MORADI, *New estimates for the numerical radius of Hilbert space operators*, *Linear Multilinear Algebra.*, **69** (5) (2021), 946–956.
- [24] M. SABABHEH, *Convexity and matrix means*, *Linear Algebra Appl.*, **506** (2016), 588–602.
- [25] M. SABABHEH, C. CONDE, AND H. R. MORADI, *A convex-block approach for numerical radius inequalities*, arXiv:2302.06777 [math.FA].
- [26] M. SABABHEH, H. R. MORADI, *More accurate numerical radius inequalities (I)*, *Linear Multilinear Algebra.*, **69** (10) (2021), 1964–1973.
- [27] M. SABABHEH, H. R. MORADI, AND Z. HEYDARBEGYI, *Buzano, Kreĭn and Cauchy-Schwarz inequalities*, *Oper. Matrices.*, **16** (1) (2022), 239–250.
- [28] S. SHEYBANI, M. SABABHEH, AND H. R. MORADI, *Weighted inequalities for the numerical radius*, *Vietnam J. Math.*, **51** (2023), 363–377.
- [29] U. YAMANCI, M. GUESBA, AND D. USLU, *Some novel inequalities for Berezin number of operators*, *Oper. Matrices.*, **18** (2) (2024), 441–455.