COMPARISON INEQUALITIES BETWEEN COMPLEX POLYNOMIALS FOR THE MAXIMUM MODULUS OF THEIR POLAR DERIVATIVE

ABDULLAH MIR

Abstract. In this paper, we prove certain comparison inequalities between two complex polynomials for the maximum modulus of their polar derivative, when the zeros of one of the polynomial are restricted. A variety of interesting results follow as special cases from our results.

Mathematics subject classification (2020): 30A10, 30C10, 30C15.

Keywords and phrases: Polar derivative of a polynomial, maximum modulus principle, Bernstein-inequality.

REFERENCES

[1] N. C. ANKENY AND T. J. RIVLIN, *On a theorem of S. Bernstein*, Pacific J. Math., **5** (1955), 849–852.

[2] S. BERNSTEIN, *Sur l'ordre de la meilleure approximation des fonctions continues par des polynômes de degré donné*, Mem. Acad. R. Belg., **4** (1912), 1–103.

[3] S. BERNSTEIN, *Sur la limitation des dérivées des polynômes*, Comptes Rendus de l' Academic des Sciences, (Paris) **190** (1930), 338–340.

[4] N. K. GOVIL, M. A. QAZI AND Q. I. RAHMAN, *Inequalities describing the growth of polynomials not vanishing in a disk of prescribed radius*, Math. Inequal. Appl., **6** (2003), 453–467.

[5] N. K. GOVIL, A. LIMAN AND W. M. SHAH, *Some inequalities concerning derivative and maximum modulus of polynomials*, Austr. J. Math. Anal. Appl., **8** (2011), 1–8.

[6] V. K. JAIN, *Inequalities for a polynomial and its derivative*, Proc. Indian Acad. Sci. (Math. Sci.), **110** (2000), 137–146.

[7] P. N. KUMAR, *On the generalization of polynomial inequalities in the complex domain*, J. Contemporary Math. Anal., **50** (2015), 14–21.

[8] A. LIMAN, R. N. MOHAPATRA AND W. M. SHAH, *Inequalities for polynomials not vanishing in a disk*, Appl. Math. Comp., **218** (2011), 949–955.

[9] A. LIMAN, I. Q. PEER AND W. M. SHAH, *On some inequalities concerning the polar derivative of a polynomial*, Ramanujan J., **38** (2015), 349–360.

[10] P. D. LAX, *Proof of a conjecture of P. Erdös on the derivative of a polynomial*, Bull. Amer. Math. Soc., **50** (1944), 509–513.

[11] M. MARDEN, *Geometry of Polynomials*, Math. Surveys, **3** (1966).

[12] H. A. S. MEZERJI, M. A. BASERI, M. BIDHKAM AND A. ZIREH, *Generalization of certain inequalities for a polynomial and its derivative*, Lob. J. Math., **33** (2012), 68–74.

[13] A. MIR, *On an operator preserving inequalities between polynomials*, Ukrainian Math. J., **69** (2018), 1234–1247.

[14] A. MIR AND I. HUSSAIN, *On the Erdös-Lax inequality concerning polynomials*, C. R. Acad. Sci. Paris Ser. I, **355** (2017), 1055–1062.

[15] A. MIR AND M. I. SHEIKH, *Some upper bound estimates for the maximal modulus of the polar derivative of a polynomial*, J. Contemporary Math. Anal., **55** (2020), 189–195.

[16] A. MIR AND A. WANI, *A note on two recent results about polynomials with restricted zeros*, J. Math. Inequal., **14** (2020), 47–52.

[17] Q. I. RAHMAN AND G. SCHMEISSER, *Analytic Theory of Polynomials*, Oxford University Press, (2002).

[18] W. M. SHAH, *A generalization of a theorem Paul Turán*, J. Ramanujan Math. Soc., **1** (1996), 67–72.