

GENERALIZATIONS OF NUMERICAL RADIUS INEQUALITIES FOR HILBERT SPACE OPERATORS

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Abstract. In this work, some generalizations of numerical radius inequalities for sums and products of Hilbert space operators are presented. These new inequalities generalize some existing inequalities given in [3] and [8].

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