ON JENSEN–TYPE INEQUALITIES FOR
HARMONIC CONVEX FUNCTIONS

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Abstract. Inequalities play a main role in pure and applied mathematics. In particular, Jensen inequality plays an important role in many fields of Mathematics. In this paper we prove two new Jensen-type inequalities for harmonic convex functions via fractional calculus, and we apply them to generalized Caputo-type fractional integrals.

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REFERENCES

[1] A. ATANGANA, D. BALEANU, *New fractional derivatives with nonlocal and non-singular kernel. Theory and application to heat transfer model*, Therm. Sci. **20** (2) (2016) 763–769, https://doi.org/10.2298/TSCI160111018A.

[2] D. BALEANU, K. DIETHELM, E. SCALAS, J. J. TRUJILLO, *Fractional Calculus: Models and Numerical Methods*, Series on Complexity, Nonlinearity and Chaos, vol. 3, Singapure: World Scientific Publishing, 2017, ISBN 9789814355209.

[3] I. A. BALOCH, A. A. MUGHAL, Y.-M. CHU, A. UL HAQ, M. DE LA SEN, *A variant of Jensen-type inequality and related results for harmonic convex functions*, AIMS Math. **5** (6) (2020) 6404–6418, doi:10.3934/math.2020412.

[4] P. BOSCH, Y. QUINTANA, J. M. RODRÍGUEZ, J. M. SIGARRETA, *Jensen-type inequalities for m-convex functions*, Open Math. **20** (2022) 946–958.

[5] P. BOSCH, J. M. RODRÍGUEZ, J. M. SIGARRETA, *Oscillation results for a nonlinear fractional differential equation*, AIMS Math. **8** (5) (2023) 12486–12505, doi:10.3934/math.2023627.

[6] P. BOSCH, J. M. RODRÍGUEZ, J. M. SIGARRETA, *On new Milne-type inequalities and applications*, J. Inequal. Appl. **2023**, 3 (2023), https://doi.org/10.1186/s13660-022-02910-0.

[7] P. BOSCH, A. PORTILLA, J. M. RODRÍGUEZ, J. M. SIGARRETA, *On a generalization of the Opial inequality*, Demonstratio Mathematica **57** (1) (2024) pp. 202301149, https://doi.org/10.1515/dema-2023-0149.

[8] M. CAPUTO, *Linear model of dissipation whose Q is almost frequency independent II*, Geophys. J. Int. **13** (5) (1967) 529–539, https://doi.org/10.1111/j.1365-246X.1967.tb02303.x.

[9] M. CAPUTO, *Elasticità e Dissipazione*, Bologna: Zanichelli, 1969.

[10] M. CAPUTO, M. FABRIZIO, *A new definition of fractional derivative without singular kernel*, Progr. Fract. Differ. Appl. **1** (2) (2015) 73–85, http://dx.doi.org/10.12785/pfda/010201.

[11] Z. DAHMANI, *On Minkowski and Hermite-Hadamard integral inequalities via fractional integral*, Ann. Funct. Anal. **1** (2010) 51–8.

[12] S. S. DRAGOMIR, *Inequalities of Jensen type for HA-convex functions*, An. Univ. Oradea Fasc. Mat. **27** (2020) 103–124.

[13] J. HAN, P. OTHMAN MOHAMMED, H. ZENG, *Generalized fractional integral inequalities of Hermite-Hadamard-type for a convex function*, Open Math. **18** (2020) 794–806.

[14] J. L. W. V. JENSEN, *Sur les fonctions convexes et les inégalités entre les valeurs moyennes*, Acta Math. **30** (1) (1906) 175–193.

[15] P. KÓRUS, J. E. NÁPOLES VALDÉS, *q-Hermite-Hadamard inequalities for functions with convex or h-convex q-derivative*, Math. Ineq. Appl. **25** (2) (2022) 601–610.

[16] P. KÓRUS, J. E. NÁPOLES VALDÉS, J. M. RODRÍGUEZ, J. M. SIGARRETA ALMIRA, *Petrović-type inequality via fractional calculus*, Miskolc Mathematical Notes **25**: 2 (2024) 819–828.

[17] A. MCD. MERCER, *A variant of Jensen's inequality*, J. Ineq. Pure Appl. Math. **4** (4) (2003) Article 73.

[18] S. MUBEEN, S. HABIB, M. N. NAEEM, *The Minkowski inequality involving generalized k-fractional conformable integral*, J. Inequal. Appl. **2019** (2019) 81.

[19] K. S. NISAR, F. QI, G. RAHMAN, S. MUBEEN, M. ARSHAD, *Some inequalities involving the extended gamma function and the Kummer confluent hypergeometric K-function*, J. Inequal. Appl. **2018** (2018) 135.

[20] G. RAHMAN, T. ABDELJAWAD, F. JARAD, A. KHAN, K. SOOPPY NISAR, *Certain inequalities via generalized proportional Hadamard fractional integral operators*, Adv. Difference Eq. **2019** (2019) 454.

[21] G. RAHMAN, K. SOOPPY NISAR, B. GHANBARI, T. ABDELJAWAD, *On generalized fractional integral inequalities for the monotone weighted Chebyshev functionals*, Adv. Difference Eq. **2020** (2020) 368.

[22] S. RASHID, M. ASLAM NOOR, K. INAYAT NOOR, Y.-M. CHU, *Ostrowski type inequalities in the sense of generalized K-fractional integral operator for exponentially convex functions*, AIMS Math. **5**: 3 (2020) 2629–2645.

[23] Y. SAWANO, H. WADADE, *On the Gagliardo-Nirenberg type inequality in the critical Sobolev-Orrey space*, J. Fourier Anal. Appl. **19** (2013) 20–47.

[24] E. SET, M. TOMAR, M. Z. SARIKAYA, *On generalized Grüss type inequalities for k-fractional integrals*, Appl. Math. Comput. **269** (2015) 29–34.

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