# A TREATMENT METHOD OF A CLASS OF HALF–DISCRETE HILBERT–TYPE INEQUALITIES ON SYMMETRIC SETS

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*Abstract.* In this work, we first construct a special set of real numbers, and then we define a new half-discrete kernel function on symmetric sets with the parameters limited to the newly constructed set. By virtue of some techniques of real analysis, we transform the weight function to the first quadrant to estimate its upper bound, then a half-discrete Hilbert-type inequality on symmetric sets is proved, and its constant factor is proved to be optimal. Furthermore, the equivalent Hardy-type inequalities of the newly obtained Hilbert-type inequality are also considered. Lastly, assigning special values to the parameters in the kernel function, some new special half-discrete Hilbert-type inequalities are provided at the end of the paper.

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