

## MULTILINEAR $\theta$ -TYPE CALDERÓN–ZYGMUND OPERATORS AND THEIR COMMUTATORS ON PRODUCTS OF WEIGHTED AMALGAM SPACES

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**Abstract.** In this paper, we first introduce several new classes of weighted amalgam spaces. Then we discuss both strong type and weak type estimates for certain multilinear  $\theta$ -type Calderón–Zygmund operators  $T_\theta$  recently introduced in the literature on products of these spaces with multiple weights. Furthermore, the strong type and weak end-point estimates for both multilinear commutators and iterated commutators of  $T_\theta$  and pointwise multiplication with BMO functions are established as well.

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