

SHARP WEIGHTED POWER MEAN BOUNDS FOR TWO LEMNISCATE TYPE MEANS

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Abstract. In this paper, we present sharp weighted power mean bounds for two lemniscate type means, which were introduced by Neuman (Math Pannon **18** (1): 77–94, 2007). As a corollary, we get two new sharp power mean bounds for two lemniscate type means.

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