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MONOTONICITY PROPERTIES OF WEIGHTED GEOMETRIC SYMMETRIZATIONS

KATARINA BOGDANOVIĆ AND ALJOŠA PEPERKO*

Abstract. We prove new monotonicity properties for spectral radius, essential spectral radius, operator norm, Hausdorff measure of non-compactness and numerical radius of products and sums of weighted geometric symmetrizations of positive kernel operators on L^2. To our knowledge, several proved properties are new even in the finite dimensional case.

Mathematics subject classification (2020): 47A10, 47B65, 47B34, 15A42, 15A60, 15B48.
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www.ele-math.com
jmi@ele-math.com