

REFINEMENTS AND APPLICATIONS OF SEVERAL HERMITE—HADAMARD TYPE INEQUALITIES

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Abstract. In the work, the authors establish several Hermite–Hadamard type inequalities, which refine those inequalities in Theorems 1 and 2 in the paper “C. E. M. Pearce and J. Pečarić, *Inequalities for differentiable mappings with application to special means and quadrature formula*, Appl. Math. Lett. **13** (2000), no. 2, 51–55”, and give some applications to special means.

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