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A NEW HALF–DISCRETE MULTIDIMENSIONAL HILBERT–TYPE INEQUALITY INVOLVING ONE HIGHER–ORDER DERIVATIVE FUNCTION

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Abstract. This paper presents a new half-discrete multidimensional Hilbert-type inequality involving one higher-order derivative function utilizing transfer formula and Hermite–Hadamard's inequality. The inequality investigates a general intermediate variable in kernel $\frac{1}{\left(x+\|v(k)\|_\alpha\right)^{\lambda+m}}$ $(x,\lambda>0)$ than previous work. The research explores the best value related to certain parameters. Finally, the equivalence forms and operator expressions are also presented.

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