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FURTHER DEVELOPMENTS OF BELLMAN AND ACZÉL INEQUALITIES FOR OPERATORS

PARVANEH ZOLFAGHARI

Abstract. In the present paper, we derive some operator Bellman and Aczél inequalities involving quasi λ-geometric and arithmetic means. Among other inequalities, it is shown that if $\Phi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a unital positive linear map and $A, B \in \mathbb{B}(\mathscr{H})$ are two contraction operators, then for any $p > 1$,

$$\Phi\left((I - A\nabla_\lambda B)^{\frac{1}{p}}\right) \leqslant \Phi(I-A)^{\frac{1}{p}} \nabla_\lambda \Phi(I-B)^{\frac{1}{p}}$$

holds, where $\lambda \notin [0,1]$.

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